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System Availability Optimization: Integrating Quantity Discounts and Delivery Lead Time Considerations

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ABSTRACT

Designing an industrial system involves a trade-off between the quality of its components, their purchasing cost, and the time needed to deliver and assemble them. Higher-quality components increase system availability but cost more, and large orders reduce unit prices through quantity-based discounts but extend delivery lead times, which may delay system construction and incur penalties. This paper proposes a model that allocates the orders of the components of a series-parallel multi-state system to alternative suppliers, considering price, reliability, repair rate, quantity-based discount, and delivery lead time simultaneously. The system is described by a continuous-time Markov chain with full-capacity, half-capacity, and shutdown states, whose steady-state probabilities are embedded in a nonlinear binary mathematical program. The program minimizes the total cost of the construction phase, including component purchase and project delay penalties, and of the exploitation phase, including the cost of operating at half capacity and of shutdown, subject to a budget limit, precedence relations among assembly activities, and a minimum system reliability. A numerical example based on the feedwater system of heat recovery steam generator boilers in combined-cycle power plants illustrates the model. The optimal plan achieves a system reliability of about 0.85, and the delay penalty accounts for more than half of the total cost. Sensitivity analyses show that removing quantity-based discounts lowers the quality of the selected components and increases operating costs, that a fixed delivery lead time leads to the selection of high-reliability components, and that the daily delay penalty is the most influential parameter. The proposed approach helps decision makers balance construction objectives and long-term operational availability.

1. Introduction

Important industries such as power plants are exposed to rapid technological developments. Given that systems in these industries have complex configurations, failure of any component can cause malfunction or severe failure. System availability is a suitable metric for evaluating the reliability of a repairable system. Repairable systems have repairable parts that can be repaired to

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restore normal operation in the event of failure [1]. The importance of designing systems with minor failures and malfunctions, and with high availability, is increasing. This is due to engineering requirements that demand higher quality and safety levels in products [2]. Availability is the most recognized terminology used to assess the efficiency of industrial systems with repairable elements [3]. As a result, it is crucial to maintain systems available and increase their availability. Systems availability shows the percentage of time systems work and function [4]. A series-parallel system consists of subsystems connected in series, while each subsystem has components connected in parallel. A subsystem fails by failing all its elements, and whenever one of these subsystems fails, the entire system fails [5]. Availability modeling, one of the main approaches to assess the availability of a system and its subsystems, tries to determine the availability of each element based on system requirements or objectives [6].

When designing a system, there are always constraints related to the available manufacturing budget and time. On the other hand, system availability impacts the expected system costs and risks during the exploitation phase. High-quality components improve the entire system's availability; however, they are more expensive. Lead times of suppliers may be different. Also, large orders benefit from the quantity-based discount; however, it leads to longer lead times because suppliers will need more time to prepare the additional components. The extended parts delivery lead times may cause delays in the system construction and result in a delayed penalty. Therefore, the design of a repairable system cannot be separated from the supplier selection decisions that determine the purchasing price, delivery lead time, and quality of its parts. The related literature is reviewed in three groups that follow the growth of knowledge in this area: reliability allocation and Markov-based availability modeling, supplier selection considering reliability, and supplier selection and order allocation with quantity discounts, lead times, and fuzzy and goal programming methods.

Early reliability optimization work relied on metaheuristics: Coit and Smith [7] used a genetic algorithm to optimize series-parallel systems, while Kuo and Wan [8] reviewed recent advances in optimal reliability allocation. For multistate systems, Ouzineb et al. [9] proposed a tabu search for the redundancy allocation problem of homogeneous series-parallel multistate systems. More recently, Ostadi and Hamedankhah [10] developed a two-stage reliability optimization approach for the series-parallel redundancy allocation problem that considers the sale of worn-out parts. Peiravi et al. [11] assessed redundancy strategies for k-out-of-n systems and optimized them using Markov chains and genetic algorithms. Li and Zhang [12] examined methods for exact solution approaches for the redundancy allocation problem in multistate series-parallel systems, and Liu et al. [13] discussed multistate system reliability as an emerging paradigm for sophisticated engineered systems. These studies show that redundancy allocation remains central to the design of repairable and multistate systems. Many researchers also applied availability and reliability analysis to optimize system configuration. Pascual et al. [14] presented a model and a heuristic procedure to optimize the equipment's availability, procurement costs, and service levels subject to a budget constraint. The model uses a single-echelon structure with repairable parts to determine the optimal number of repairable stocked spare parts, and the results show that an optimal procurement policy of spare parts produces better availability.

Since the repair time of components is not negligible, a continuous-time Markov chain is a suitable stochastic tool to describe and analyze system behavior under normal and failure conditions. It generates equations representing the portion of time the system stays in each state [15]. Billinton and Lee [16] addressed an approach for reliability modeling utilizing the Markov approach for evaluating the design of a heat transport pump. Using a continuous-time Markov chain model, Kumar et al. [17] analyzed the chain-based scheme's availability and reliability. Che et al. [18] introduced a Markov chain-based availability model for studying the availability of virtual cluster systems. This

model provides the possibility of explaining the lifecycle state and state transition of the virtual cluster node, and they also obtained the availability level of each virtual cluster node in a complicated virtual cluster system or cloud data center. Abbas et al. [19] estimated the performance of a degraded multi-state system with three failure states using a recursive Markov chain closed-form and an analytical solution considering a condition-based maintenance model. Huang et al. [20] employed a Markov-chain-based model to assess the availability of wind turbines. They used a Poisson-process-based algorithm to estimate systems' state transition rates and solved a transition matrix to calculate expected availability.

Siddiqui et al. [21] optimized the availability of a multi-state series system configuration using the Markov model. They developed the optimal maintenance plan and validated the Monte Carlo simulation results. Mooren [22] addressed a new approach to making preliminary decisions for system design. He utilized the Markov model and a network-theory-based approach to maximize the system availability of a naval vessel. Kumar et al. [23] used a semi-Markov process model to analyze the steady-state availability of mechanical systems that follow condition-based maintenance and evaluate optimal condition monitoring intervals. They used this methodology to analyze maintenance policies for a centrifugal pump. The application of Markov chains has also expanded to other domains. Tang et al. [24] benefited from Markov chain concepts in a service-based network model to design an availability evaluation approach for complex networks. Their method determines the minimum performance cost based on the quality-of-service metrics. Afsharnia et al. [25] used the Markov chain method to evaluate the availability of sugarcane harvester machines in the agro-industries. Sun et al. [26] proposed a model to describe correlated fading processes in dual-frequency global navigation satellite system signals. They applied a Markov chain with stationary conditional probabilities based on the Poisson rate parameters by representing transitions in four fading states.

Markov chain-based availability has been widely used in reliability-based design optimization (RBDO) to model system performance under uncertainty, degradation, and failure. This approach helps the analytical estimation of the steady-state or transition probability between system states, thereby calculating the proportion of time a system stays in operational, degraded, or failed states. Prior studies have illustrated the effectiveness of Markov models in showing component dependencies, repairs and replacements, and state-transition uncertainty, which make them particularly suitable for complex industrial systems [27]-[35]. Khojaste et al. [36], [37] applied Markov decision process models to optimize operational availability in energy systems, including electricity grids and wind-driven generation, demonstrating the use of stochastic state transitions to improve reliability and cost efficiency. Moreover, data-driven approaches based on time-inhomogeneous Markov models and stochastic optimization have the potential to address RBDO more precisely, particularly when system dynamics exhibit seasonal or cyclic behavior [38], [39], [36]. These works highlight how availability-based performance metrics derived from Markov processes can be integrated into optimization frameworks to support design decisions that balance reliability, cost, and system lifecycle considerations. However, in these studies the quality and price of the components are mostly treated as given inputs, although they are determined by the supplier selection decisions.

Traditional supplier selection methods often consider the purchase and shipping costs, ignoring the reliability and maintainability of purchased parts [40]. To overcome this limitation, researchers started to add reliability to the supplier selection criteria. Jiao et al. [41] proposed a new supplier house of quality approach consisting of customer requirements and supplier reliability and designability. They also addressed the mathematical programming model of the supplier quality house to resolve the synthesis parts' weighted scale. Kanagaraj and Jawahar [42] offered a reliability-based model to minimize procurement, maintenance, and downtime costs subject to the practical

constraints on product reliability and weight limitation and solved the model by the simulated annealing algorithm. Tomaru et al. [43] introduced new measures for supplier quality evaluation and a systematic approach to select suppliers. Kanagaraj et al. [40] proposed a model for the supplier selection problem using the reliability-based total cost of ownership approach regarding reliability and cost factors, including the product's initial price, quality, service, and maintenance-related costs. They also developed a meta-heuristic optimization algorithm by combining cuckoo search with a genetic algorithm to solve the problem.

Subsequent studies linked supplier selection more directly to the availability of the system. Hague et al. [44] considered the selection of suppliers based on their ability to provide parts that improve the system's availability. They employed the availability of individual components as a criterion for the decision problem and weighted those criteria based on component importance and described the reliability and maintainability parameters by interval numbers. Yang [45] also worked on the supplier selection problem. He employed operational cost and reliability factors to assess a supplier and then solved the model using a quantitative approach. Rajabi Asadabadi [46] presented a Markov chain to find a pattern for the changing preferences of customer needs. A combination of the analytic network process and quality function deployment links this pattern to product requirements and product requirements to supplier qualifications. Azarian et al. [47] explained eight essential practices, including reliability requirements and planning, training and development, reliability analysis, reliability testing, supply chain management, failure data tracking and analysis, verification and validation, and reliability improvement for supplier assessment problems.

Beyond reliability, the order quantity affects the price and delivery time of the components, which has been considered in the broader supplier selection and order allocation literature. Ho et al. [48] reviewed multi-criteria decision-making approaches for supplier evaluation and selection, and Aissaoui et al. [49] reviewed supplier selection and order lot sizing models. Some researchers studied the effects of the order quantities on the component parts and used them in supplier selection problems. Qin et al. [50] considered an all-units quantity-based discount in a freight allocation problem. They used a heuristic-based algorithm combining a filter-and-fan and a tabu search mechanism. Hamdan and Cheaitou [51] worked on a multi-period green supplier selection and order allocation problem. They developed a single-product bi-objective integer linear programming model with deterministic demand, considering all-unit quantity discounts. Pourmohammadreza et al. [52] reviewed discount policies in supplier selection, and Badi et al. [53] addressed efficient scheduling and delivery optimization in vendor-managed inventory. Gharibi et al. [54] integrated real-world customer data and road networks within a mixed integer linear programming approach for sustainable industrial hemp value chains.

Because supplier selection involves several conflicting criteria and imprecise information, fuzzy and goal programming methods have been combined with it. Fuzzy methods are common in this field, starting from the fuzzy programming approach of Zimmermann [55]. Kamran et al. [56] used goal programming to develop a resource allocation model for a healthcare emergency center, and Masoumzadeh et al. [57] proposed a multi-objective fuzzy goal programming model for P-hub location and protection with back-up hubs. Çebi and Otay [58] proposed a two-stage fuzzy approach for supplier evaluation and order allocation with quantity discounts and lead time. Safaeian et al. [59] developed a multi-objective fuzzy model for supplier selection and order allocation under incremental discounts. Gupta et al. [60] studied a multi-objective vendor selection problem in a fuzzy environment. Eslamipoor [61] proposed a fuzzy multi-objective model for supplier selection that mitigates the impact of vehicle transportation gases and delivery time, and Keshavarz-Ghorabaee [62] addressed sustainable supplier selection and order allocation using an integrated ROG-based type-2 fuzzy decision-making approach. Closer to the topic of this paper, Shahrokhi et al. [31] used a

goal programming approach for supplier selection, and Sobhani et al. [34] applied fuzzy goal programming to availability-based design optimization.

A literature review shows that reliability allocation and Markov-based availability models have rarely been combined with the supplier selection decisions that determine the price, quality, and delivery lead time of the components, and that researchers did not consider optimizing system availability regarding component quantity-based discounts and delivery lead time. This gap is important in power plants and other essential industries, where late deliveries delay the system construction and the unavailability of the system during operation is costly. This paper aims to minimize the product's life cycle cost, including the cost of building and operation, by considering the budget constraints and a minimum required reliability level. This paper presents a model to construct a specific series-parallel multi-state industrial system concerning availability, purchase price, and system construction delay cost, and to optimize the feedwater system of heat recovery steam generator (HRSG) boilers used in combined cycle power plants. Due to the non-negligible repair time, this paper uses the results of the continuous Markov chain approach in a nonlinear binary program. This binary mathematical program optimizes the system's costs during its construction and operating phases simultaneously. The following section defines the problem and develops a simplified model for optimizing the feedwater system of HRSG boilers, and the final sections interpret and discuss the results of the numerical example.

2. Methodology

2.1 Problem Statement

This study concerns the selection of components for a part of the boilers' feedwater system used in power plants. It is a multi-state series-parallel system composed of the subsystems shown in Figures 1 and 2.

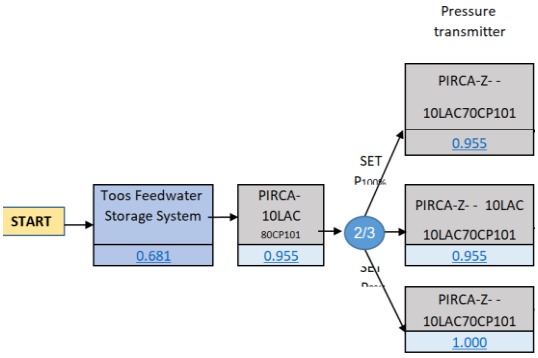


Fig. 1. The reliability block diagram of the studied series-parallel system

Generally, the suppliers of two serial components are identical. Combining these components into part A, as shown in Figure 2, simplifies the model.

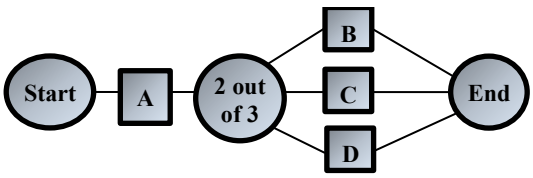


Fig. 2. Simplified studied series-parallel system

Several alternative suppliers offer different prices, qualities (failure and repair rates), and delivery lead times. The availability of the entire system depends on the reliability and repair rates of its subsystems, so the reliability, repair rate, price, and delivery lead time of the components are the criteria used to allocate orders to the suppliers. Despite quantity-based discounts, large orders increase the delivery lead time when several parts are ordered from a single supplier. Each of B, C, and D can provide 50% of the nominal capacity. Therefore, the system states are: (1) full capacity (FC), when A operates and at least two out of B, C, and D are operational; (2) half capacity (HC), when A operates and only one out of B, C, and D is working; and (3) shutdown (SD), when A or all of B, C, and D are not functional. The HC and SD states cause lost production and repair costs. On the one hand, more reliable components increase the availability of the entire system; on the other hand, they are generally more expensive and have longer delivery lead times. In addition, there is a daily penalty for delays in the system completion time. Therefore, the objective of the model is to minimize the total cost of system assembly and exploitation.

The Markov chain model represents a class of stochastic processes in which the probability of a future event depends only on the present state [63], [64]. As the process evolves, the system can remain in the same state or move to a different one. Transition probabilities describe the likelihood of these movements. As the following equation shows, the transition matrix (Markov matrix) of a system with M states contains all the transition probabilities (P_{ij}), which give the likelihood of changing the system state from i to j.

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ p_{M1} & p_{M2} & \cdots & p_{MM} \end{bmatrix} \quad (1)$$

All the matrix elements are nonnegative, and the sum of the numbers in each row equals 1. A state-space diagram shows the system states, transition paths, and related transition rates. Figure 4 shows the state-space diagram of the feedwater system used in HRSG boilers. The nodes in this diagram represent the system states, and the branches represent the transitions from one state to another. The values on the branches indicate the transition rates, which may be λ or μ, meaning the system failure and repair rates, respectively [65]. The following sections present the model assumptions, indices, parameters, objective function, and constraints.

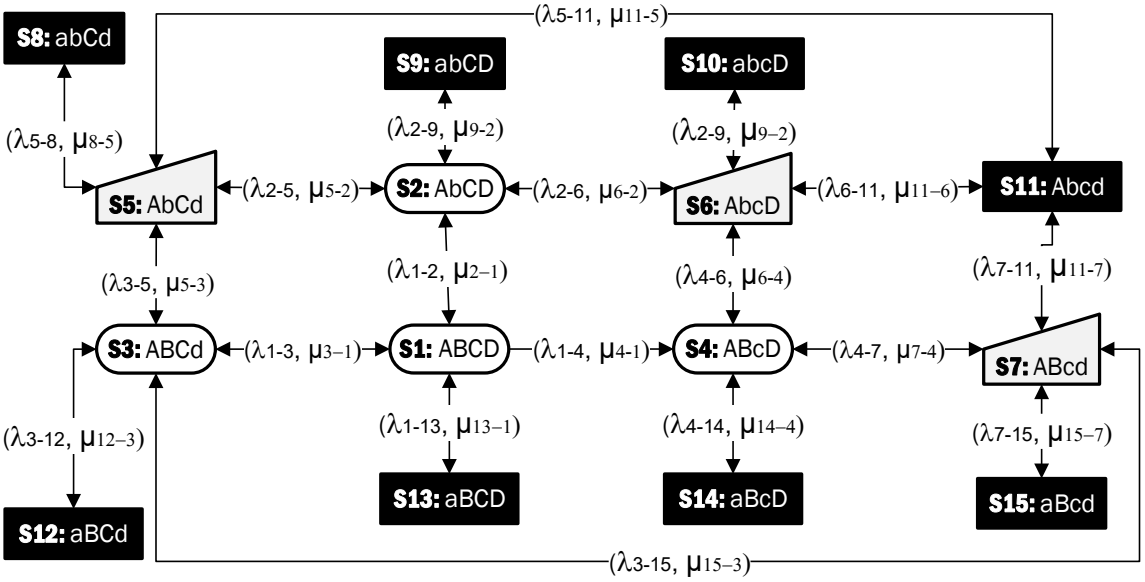


Figure 4. The system Markov chain model

2.2 Assumptions

This study uses a continuous-time Markov chain to obtain the probability of each state, where the time that the system spends in each state before moving to the next state is an independent exponential random variable [64]. The other assumptions of the model are as follows:

1. The construction project has a deadline, and any delay imposes a fixed daily cost on the project.
2. There are several alternative suppliers for each component.
3. The time and cost of the component assembly processes are independent of the quality of the components.
4. Suppliers offer components with known quality (reliability and repair time).
5. The delivery lead time and price offered by a supplier depend on the order quantity.
6. Components B, C, and D are identical in function, and each provides 50% of the nominal system capacity.
7. The system is always in one of three states: (1) full capacity (FC), (2) half capacity (HC), and (3) shutdown (SD), according to the failure or working of its components.
8. There are daily production loss penalties for the HC and SD states.
9. The components' mean time to repair is not negligible.
10. The system has an infinite lifetime.
11. The failure and repair rates of the components are constant during the planning horizon.
12. The purchase price includes the costs of ordering, shipment, and assembly of the components.
13. The construction of B, C, and D begins after the assembly of A is completed.
14. The minimum attractive rate of return (MARR) is known.

2.3 Indices

$i = 1,2,3,4$ The components' index (i=1, 2, 3, and 4 for A, B, C, and D, respectively)
$j = 1,2, \dots, J$ The suppliers' index
$k = 1,2, \dots, K$ Index of states of the system

2.4 Parameters

C_{1j}: The purchase price of component A from supplier j
C_{ij}: The unit purchase price of B, C, or D from supplier j, when ordered separately
C'_{ij}: Unit purchase price when two components out of B, C, or D ordered to supplier j
$C^{"}_{ij}$: Unit purchase price when B, C, and D ordered to supplier j
R_{ij}: Reliability of component i from supplier j
R: The requested minimum system's reliability
T: System construction deadline
B: The maximum available budget for purchasing components
C_2: The daily cost of SD
C_3: The daily cost of working the system at half capacity
C^D: Daily delay penalty for the system construction
L_{Aj}: The delivery lead time of component A from supplier j
L_{Bj}: The delivery lead time of components B, C, or D from supplier j, when ordered separately
L'_{Bj}: The delivery lead time, when two components of B, C, or D ordered to supplier j
$L^{"}_{Bj}$: The delivery lead time when all B, C, and D, ordered to supplier j
F_{ik}: Time of the process k on component i
K_1: Set of the required processes for construction component 1
K_2: Set of the required processes for construction components 2, 3 and 4
r: The minimum acceptable rate of return (MARR)
α: The hourly cost of SD
β: The hourly cost of HC
μ_{ij}: Repair rate of component i from supplier j expressed in terms of the average number of performed repairs actions per hour
λ_{ij}: The failure rate of component i from supplier j (expressed in terms of the average number of failures per hour)
A_1: Total purchase cost
A_2: The expected total cost of the system shutting down (in the entire life cycle of the system)
A_3: The total cost of the working system at half capacity (in the entire life of the system)
A_4: Total system construction delay penalty
y_{ij}: A binary variable; equal to 1 if component i is purchased from supplier j and zero otherwise
R_i: Reliability of component i
R_e: Reliability of the entire system
P_0: The proportion of the time that the system is shutting down
P_{50}: The proportion of the time that the system is working at half capacity
S_k: The proportion of the time that the system is in state k
μ_i: Repair rate of component i (expressed in terms of the average number of repairs actions performed per hour)
λ_i: The failure rate of component i (expressed in terms of the average number of failures per hour)

T^c: Completion time of the system construction project
T_c^i: Completion time of construction component i
L: Required time for delivering all components B, C, and D

2.5 Mathematical Model

This study develops an approach for determining the optimal configuration of a typical industrial system, and the same method is applicable to other cases. The binary nonlinear programming (BNLP) model is as follows. The objective of the model is to minimize the expected total cost of the system, including the purchase cost, the project construction delay penalty, and the cost of being in the shutdown and half-capacity states during its exploitation.

$$Min\ Z = A_1 + A_2 + A_3 + A_4 \tag{1}$$

Constraint (2) uses binary variables to calculate the purchase cost, which equals the sum of the purchase prices of all components, taking the quantity-based discounts into account.

$$\begin{aligned} A_1 = & \sum_{j=1}^{3} 3 y_{2j} y_{3j} y_{4j} C_{ij}^{''} \\ & + \sum_{j=1}^{3} \begin{pmatrix} y_{2j} y_{3j} \left(1 - y_{4j}\right) \\ + y_{2j} y_{4j} \left(1 - y_{3j}\right) \\ + y_{3j} y_{4j} \left(1 - y_{2j}\right) \end{pmatrix} \times C_{ij}^{'} \\ & + \sum_{j=1}^{3} \begin{pmatrix} y_{2j} \left(1 - y_{3j}\right)\left(1 - y_{4j}\right) \\ + y_{3j} \left(1 - y_{2j}\right)\left(1 - y_{4j}\right) \\ + y_{4j} \left(1 - y_{2j}\right)\left(1 - y_{3j}\right) \end{pmatrix} \times C_{ij} \\ & + \sum_{j=1}^{3} y_{1j} C_{1j} \end{aligned} \tag{2}$$

Constraint (3) gives the total cost of system shutdown during its infinite exploitation period, considering the capital rate of return.

$$A_2 = 8760^* \propto P_0 * 1/r \tag{3}$$

Equation (4) calculates the proportion of time the system spends in the shutdown states, that is, when part A or all three components B, C, and D fail.

$$\begin{aligned} P_0 = & S_8 + S_9 + S_{10} + S_{11} \\ & + S_{12} + S_{13} + S_{14} + S_{15} \end{aligned} \tag{4}$$

Constraint (5) gives the total cost of operating at half capacity (when the system works at 50% of its power), considering the capital rate of return during the infinite exploitation period.

$$A_3 = 8760 * \beta P_{50} * 1/r \tag{5}$$

Constraint (6) gives the delay penalty of the system construction project. If the project is completed before the deadline ($T^c \leq T$), the delay penalty equals zero.

$$A_4 = \max\left(0, T^c - T\right) C^D \tag{6}$$

Equation (7) calculates the completion date of the installation of component A, which is the sum of its lead time and its installation time, a constant amount.

$$T_c^1 = \sum_{j=1}^{3} y_{1j} L_{Aj} + \sum_{k \in k_1} F_{1k} \tag{7}$$

Formula (8) calculates the completion date of the installation of B, C, and D. If the lead time (L) of a component is longer than the completion date of component A, its completion date is the sum of its lead time and its installation time. Otherwise, if component A is installed after the lead time of the component, T_c^i equals the completion date of component A plus the installation time of part i.

$$T_c^i = \max\left(L, T_c^1\right) + \sum_{k \in k_2} F_{ik} \qquad i = 2,3 and\ 4 \tag{8}$$

Constraint (9) gives the delivery lead time of components B, C, and D according to their suppliers.

$$\begin{aligned} L = & \sum_{j=1}^{3} 3 y_{2j} y_{3j} y_{4j} L''_{Bj} \\ & + \sum_{j=1}^{3} \begin{pmatrix} y_{2j} y_{3j} \left(1 - y_{4j}\right) \\ + y_{2j} y_{4j} \left(1 - y_{3j}\right) \\ + y_{3j} y_{4j} \left(1 - y_{2j}\right) \end{pmatrix} \\ & \times Max\left(L'_{Bj}, L_{Bj}\right) \\ & + \sum_{j=1}^{3} \begin{pmatrix} y_{2j} \left(1 - y_{3j}\right)\left(1 - y_{4j}\right) \\ + y_{3j} \left(1 - y_{2j}\right)\left(1 - y_{4j}\right) \\ + y_{4j} \left(1 - y_{2j}\right)\left(1 - y_{3j}\right) \end{pmatrix} \\ & \times L_{Bj} \end{aligned} \tag{9}$$

Equation (10) gives the completion date of the project construction.

$$T_c = \max\left(T_c^i\right) \tag{10}$$

Equation (11) calculates the proportion of time the system spends in the half-capacity states (states 5, 6, and 7).

$$P_{50} = S_5 + S_6 + S_7 \tag{11}$$

Constraint (12) limits the cost of purchasing the components to the available budget.

$$A_1 \leq B \tag{12}$$

Equations (13), (14), and (15) give the reliability, repair rate, and failure rate of each component, respectively, according to the selected supplier.

$$R_i = \sum_{j=1}^{3} Y_{ij} R_{ij} \qquad \forall i \tag{13}$$

$$\mu_i = \sum_{j=1}^{3} Y_{ij} \mu_{ij} \qquad \forall i \tag{14}$$

$$\lambda_i = \sum_{j=1}^{3} Y_{ij} \lambda_{ij} \qquad \forall i \tag{15}$$

Constraint (16) ensures that only one supplier is selected for each component.

$$y_{ij} \in \{0.1\} \tag{16}$$

Equation (17) defines the variables y_{ij} as binary variables.

$$\sum_{j=1}^{3} y_{ij} = 1 \qquad \forall i \tag{17}$$

Equations (18) to (32) are the equilibrium equations of the continuous Markov process model. They are written according to Figure 4 and ensure that the input flows of each state are equal to its output flows.

$$(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)S_1 = \mu_2 S_2 + \mu_4 S_3 + \mu_3 S_4 + \mu_1 S_{13} \tag{18}$$

$$(\lambda_1 + \mu_2 + \lambda_3 + \lambda_4)S_2 = \lambda_2 S_1 + \mu_4 S_5 + \mu_3 S_6 + \mu_1 S_9 \tag{19}$$

$$(\lambda_1 + \mu_4 + \lambda_3 + \lambda_2)S_3 = \lambda_4 S_1 + \mu_2 S_5 + \mu_1 S_{12} + \mu_3 S_7 \tag{20}$$

$$(\lambda_1 + \mu_3 + \lambda_2 + \lambda_4)S_4 = \lambda_3 S_1 + \mu_4 S_7 + \mu_2 S_6 + \mu_1 S_{14} \tag{21}$$

$$(\lambda_1 + \mu_2 + \lambda_3 + \mu_4)S_5 = \lambda_4 S_2 + \lambda_2 S_3 + \mu_3 S_{11} + \mu_1 S_8 \tag{22}$$

$$(\lambda_1+\mu_2+\lambda_4+\mu_3)S_6 = \lambda_3S_2+\lambda_2S_4+\mu_4S_{11}+\mu_1S_{10} \quad (23)$$

$$(\lambda_1+\mu_3+\lambda_2+\mu_4)S_7 = \lambda_4S_4+\lambda_3S_3+\mu_2S_{11}+\mu_1S_{15} \quad (24)$$

$$\mu_1S_8 = \lambda_1S_5 \quad (25)$$

$$\mu_1S_9 = \lambda_1S_2 \quad (26)$$

$$\mu_1S_{10} = \lambda_1S_6 \quad (27)$$

$$(\mu_2+\mu_3+\mu_4)S_{11} = \lambda_4S_6+\lambda_2S_7+\lambda_3S_5 \quad (28)$$

$$\mu_1S_{12} = \lambda_1S_3 \quad (29)$$

$$\mu_1S_{13} = \lambda_1S_1 \quad (30)$$

$$\mu_1S_{14} = \lambda_1S_4 \quad (31)$$

$$\mu_1S_{15} = \lambda_1S_7 \quad (32)$$

Constraint (33) ensures that the sum of the probabilities of all system states equals 1.

$$\sum_{i=1}^{15} S_i = 1 \quad (33)$$

Constraint (34) specifies the reliability of the entire system.

$$R_e = 1 - P_0 \quad (34)$$

Constraint (35) ensures that the reliability of the entire system is at least at a predefined level.

$$R_e \geq R \quad (35)$$

3. Results

3.1 Numerical Example

The Markov diagram in Figure 4 shows the set of all possible states and their relationships. In this figure, oval, trapezoid, and rectangular shapes represent the FC, HC, and SD states, respectively. Table 1 shows the values of the basic parameters of the numerical example.

Table 1
The values of basic parameters of the numerical example

Parameter	Value	Parameter	Value
$\boldsymbol{C2}$	80000	μ_{i2}	0.07
$\boldsymbol{C3}$	30000	μ_{i3}	0.1
$\boldsymbol{C^D}$	300	λ_{i1}	0.05

$\boldsymbol{B}$	1100	λ_{i2}	0.03
$\boldsymbol{T}$	68	λ_{i3}	0.01
$\boldsymbol{\alpha}$	0.2	R_{i1}	0.9
$\boldsymbol{\beta}$	0.1	R_{i2}	0.95
$\boldsymbol{r}$	0.1	R_{i3}	0.99
μ_{i1}	0.05	R	0.8
$\boldsymbol{i}$=1,2,3,4			

Table 2 shows the component prices offered by different suppliers according to the order quantity.

Table 2
The price of components

Component price		Supplier 1	Supplier 2	Supplier 2
C_{1j}		200	220	240
$\boldsymbol{i}$=2,3,4	C_{ij}	300	340	380
	C'_{ij}	250	280	320
	C''_{ij}	200	240	280

Table 3 shows the time required for the manufacturing processes.

Table 3
Duration of manufacturing processes

	$\boldsymbol{F_{i1}}$	$\boldsymbol{F_{i2}}$	$\boldsymbol{F_{i3}}$	$\boldsymbol{F_{i4}}$	$\boldsymbol{F_{i5}}$
F_{1k}	3	5	7	4	2
F_{ik} $i = 2.3.4$	6	13	16	5	-

Table 4 presents the delivery lead times of the components according to the order quantity.

Table 4
The components' delivery lead time

	Supplier 1	Supplier 2	Supplier 2
L_{Aj}	5	7	17
L_{ij}	6	19	31
L'_{Bj}	8	24	37
L''_{Bj}	12	30	42

The second column of Table 5 shows the optimal solution, including the variables and their values. In this solution, the reliability of component A is higher than that of the other components, and components B and D have the same reliability. The overall reliability of the system is about 0.85; the system is shut down 15.3% of the time and works at half capacity 29.7% of the time. The project delay is 24 days, and the delay penalty is a decisive parameter, accounting for more than half of the total cost. The remaining cost is due to component failures and their purchase price.

3.2 Sensitivity Analysis

This section explains the sensitivity of the optimal solution to variations in the model parameters.

3.2.1 The effects of quantity-based discount

The third column of Table 5 shows the result of solving the model without quantity-based discounts. In this case, the model has no feasible solution because, without price reductions, achieving the requested minimum system reliability (0.8) requires at least 1140 dollars to buy high-quality components. Even with a budget of 1200 dollars, the price of the high-quality components increases, and the model selects lower-reliability components, which increases the failure costs (i.e., half-capacity and shutdown costs) due to the reduction of the entire system's availability. On the other hand, selecting components with shorter delivery lead times reduces the project completion time and, consequently, the delay penalty. In general, removing the quantity-based discounts increases the purchase and operating costs and reduces the project delay penalty.

Table 5
The optimal solution including variables and their value

Variable (1)	Optimal Solution (2)	Sensitivity Analyses					
		Removing discount (3)	fixed delivery lead time (4)	B=1200 (5)	B=1200 & C^D=100 (6)	B=1200 and C^D=50 (7)	B=1200 & C^D=500 (8)
R_1	0.99	0.99	0.99	0.99	0.99	0.99	**0.99**
R_2	0.9	0.9	0.95	0.9	0.95	0.95	**0.9**
R_3	0.95	0.9	0.95	0.95	0.9	0.95	**0.95**
R_4	0.9	0.9	0.95	0.9	0.9	0.99	**0.9**
Z	13572	13284.1	7453.5	13572	8772.1	7247.6	**18372**
R_e	0.847	0.805	0.887	0.847	0.847	0.9	**0.847**
T^c	92	87	78	92	92	136	**92**
A_1	1080	1140	960	1080	1080	1180	**1080**
A_2	2686.13	3423.5	1984.6	2686.13	2686.13	1711.3	**2686.13**
A_3	2605.95	3020.7	1508.8	2605.95	2605.95	956.35	**2605.95**
A_4	7200	5700	3000	7200	2400	3400	**12000**
P_0	0.153	0.195	0.113	0.153	0.153	0.098	**0.153**
P_{50}	0.297	0.345	0.172	0.297	0.297	0.109	**0.297**
P_{100}	0.549	0.46	0.714	0.549	0.549	0.793	**0.549**

3.2.2 The effects of delivery lead time

Column 4 of Table 5 shows the results of solving the model with a fixed delivery lead time, independent of the order size. The model selects components B, C, and D with similar high reliability (0.95) without a radical cost increase. Therefore, the overall availability improves, which reduces the HC and SD costs. Although the components are ordered from suppliers with longer delivery lead times, the project completion time decreases. The total cost also decreases as a result of the reduction in failure costs and in the delay penalty.

3.2.3 The effects of available budget and project delay penalty

As shown in column 5 of Table 5, the optimal solution does not change when the budget increases to 1200 dollars. In other words, even with more funds to purchase components, the model does not select higher-quality components to reduce the long delivery lead time. This means that the reduction in HC and SD costs is less significant than the increase in the delay penalty.

According to columns 6 and 7, reducing the daily project delay penalty to 100 $/day does not affect the components' ordering plan and, consequently, the reliability and availability of the entire system. Therefore, the purchasing, half-capacity, and shutdown costs do not change.

However, the quality of the components increases after the daily project penalty is reduced to 50 $/day. In this case, the availability of the system increases while the system failure costs decrease. Compared with the optimal solution, the project completion time increases significantly (44 days), leading to a 1.5-fold increase in the project delay time. Therefore, in this example, a slight reduction in the delay penalty rate does not significantly affect the components' order plan, whereas a reduction to 50 $/day does. After the delay penalty is increased to 500 $/day, higher-quality components are no longer economical, so the purchase and operating costs remain constant and only the delay penalty increases.

3.3 Discussion

This paper considers a reliability allocation problem regarding supply chain parameters, including the price, reliability, and lead time of the components. The proposed model is applicable in the three-dimensional concurrent engineering context, as it simultaneously considers the component supply, system construction, and exploitation phases. The model allocates the orders of the components of a typical multi-state series-parallel system to suppliers. Developing a similar model for other systems requires redrawing the reliability block diagram and rewriting the Markov process equations. The proposed approach is applicable in various industries to improve the availability of systems and reduce the final price of products.

The model considers the following financial, operational, and quality parameters of the suppliers during the design, construction, and exploitation of the system:

- Economic parameters include the price of the components and the quantity-based discount.
- Construction parameters are the delivery lead times and the component assembly times, which affect the system construction date and the project delay penalty.
- Operational parameters are the component failure and repair rates, which affect the duration of the HC and SD states.

The model also makes the following assumptions, whose relaxation offers directions for future research:

  - The failure and repair rates of the components are assumed to be known and fixed during the life cycle of the production system. In other words, the times between failures and the repair times are random variables with an exponential distribution. Many studies use these assumptions, which are essential for developing the Markov model, and a constant failure rate is a commonly used assumption for the mature phase of the system's exploitation.

- Considering the repair rate of the components as a normal random variable would improve the model. In this case, the application of the Markov process to this problem would require simplifying assumptions.
- An infinite lifetime is acceptable for the typical feedwater system of power plant boilers. Considering a known, limited lifetime requires adapting the engineering economic factor in the objective function, which does not change the generality of the model. However, some systems have a finite lifetime, so it is helpful to evaluate and improve the model considering a limited lifetime.
- The HC and SD cost functions are independent of the duration of these states. Defining these cost functions in terms of both the HC and SD times and their durations would improve the model.
- The reliability, price, and lead time factors are constant in this paper. Future research can enhance the model by considering uncertainty in the reliability, cost, and lead times of the components.
- The effect of repair rate increments caused by the similarity of elements (i.e., batch orders) has not been investigated and is also a suggestion for future research.

4. Conclusions

This paper develops a nonlinear binary programming model to optimize the construction and exploitation costs of a multi-state parallel-series system by determining the components' ordering plan. The model minimizes the total cost of the system, including the component purchase price, the system construction delay penalty, the cost of reduced system capacity, and the shutdown cost. The model considers the effects of the components' order size on their buying price and delivery lead time. In addition, the paper studied the sensitivity of the optimal solution to the following parameters:

1- quantity-based discount
2- variable delivery lead time
3- available budget
4- the daily penalty rate

The sensitivity analyses of the numerical example showed that:

1- Reducing the delay penalty rate to 50 $/day has the most significant effect on the order plan and, as a result, on all variables of the model.

2- Removing the quantity-based discounts made the model choose components with lower quality, whereas a fixed delivery lead time and a delay penalty reduction to 50 $/day led to the selection of high-quality components.

3- Increasing the available budget and changing the daily penalty rate to 100 and 500 $/day did not affect the results.

The delay in the example system construction project significantly affects its component orders. The similarity discount of the components reduces the required initial budget and increases the availability of the entire system. The results also show the effects of variations in the delivery lead times, similarity, and purchase budget of the components on the total system cost. Future research can extend the model by considering uncertainty in the reliability, cost, and lead time parameters, limited system lifetimes, and the effect of batch orders on repair rates.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Juang, Y.-S., Lin, S.-S., & Kao, H.-P. (2008). A knowledge management system for series-parallel availability optimization and design. *Expert Systems with Applications, 34*(1), 181–193. https://doi.org/10.1016/j.eswa.2006.08.023

[2] Castro, H. F., & Cavalca, K. L. (2003). Availability optimization with genetic algorithm. *International Journal of Quality & Reliability Management, 20*(7), 847–863.

[3] Murty, A. S. R., & Naikan, V. N. A. (1995). Availability and maintenance cost optimization of a production plant. *International Journal of Quality & Reliability Management, 12*(2), 28–35. https://doi.org/10.1108/02656719510080596

[4] Yusuf, I. (2014). Comparative analysis of profit between three dissimilar repairable redundant systems using supporting external device for operation. *Journal of Industrial Engineering International, 10*(4), 199–207. https://doi.org/10.1007/s40092-014-0077-3

[5] Hu, L., Yue, D., & Li, J. (2012). Availability analysis and design optimization for a repairable series–parallel system with failure dependencies. *International Journal of Innovative Computing, Information and Control, 8*(10), 6693–6705.

[6] Chiang, C.-H., & Chen, L.-H. (2007). Availability allocation and multi-objective optimization for parallel-series systems. *European Journal of Operational Research, 180*(3), 1231–1244. https://doi.org/10.1016/j.ejor.2006.04.037

[7] Coit, D. W., & Smith, A. E. (1996). Reliability optimization of series-parallel systems using a genetic algorithm. *IEEE Transactions on Reliability, 45*(2), 254–260. https://doi.org/10.1109/24.510811

[8] Kuo, W., & Wan, R. (2007). Recent advances in optimal reliability allocation. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans, 37*(2), 143–156. https://doi.org/10.1109/TSMCA.2006.889476

[9] Ouzineb, M., Nourelfath, M., & Gendreau, M. (2008). Tabu search for the redundancy allocation problem of homogenous series-parallel multi-state systems. *Reliability Engineering & System Safety, 93*(8), 1257–1272. https://doi.org/10.1016/j.ress.2007.06.004

[10] Ostadi, B., & Hamedankhah, R. (2021). A two-stage reliability optimization approach for solving series-parallel redundancy allocation problem considering the sale of worn-out parts. *Annals of Operations Research, 304*(1), 381–396. https://doi.org/10.1007/s10479-021-04093-1

[11] Peiravi, A., Nourelfath, M., & Zanjani, M. K. (2022). Redundancy strategies assessment and optimization of k-out-of-n systems based on Markov chains and genetic algorithms. *Reliability Engineering & System Safety, 221*, 108277. https://doi.org/10.1016/j.ress.2021.108277

[12] Li, Y.-F., & Zhang, H. (2022). The methods for exactly solving redundancy allocation optimization for multi-state series-parallel systems. *Reliability Engineering & System Safety, 221*, 108340. https://doi.org/10.1016/j.ress.2022.108340

[13] Liu, Y., Xiahou, T., Zhang, Q., Xing, L., & Huang, H.-Z. (2024). Multi-state system reliability: An emerging paradigm for sophisticated engineered systems. *Frontiers of Engineering Management, 11*(3), 568–575. https://doi.org/10.1007/s42524-024-0140-8

[14] Pascual, R., Santelices, G., Lüer-Villagra, A., Vera, J., & Mac Cawley, A. (2017). Optimal repairable spare-parts procurement policy under total business volume discount environment. *Reliability Engineering & System Safety, 159*, 276–282. https://doi.org/10.1016/j.ress.2016.10.034

[15] Vasconcelos, E. M. (2017). *Steady state availability general equations of decision and sequential processes in continuous time Markov chain models* (arXiv:1701.06415). arXiv. https://doi.org/10.48550/arXiv.1701.06415

[16] Billinton, R., & Lee, S. Y. (1975). *Availability analysis of a heat transport pump configuration using Markov models* (Report No. SRD-R-41). United Kingdom.

[17] Kumar, R. A., Hareesh, K., Ganesan, K., & Manjaiah, D. H. (2009). Maximizing the availability and reliability of videos in VoD system using Markov chain. In *Second International Conference on Communication Theory, Reliability, and Quality of Service* (pp. 15–19). https://doi.org/10.1109/CTRQ.2009.16

[18] Che, J., Zhang, T., Lin, W., & Xi, H. (2011). A Markov chain-based availability model of virtual cluster nodes. In *Seventh International Conference on Computational Intelligence and Security* (pp. 507–511). https://doi.org/10.1109/CIS.2011.118

[19] Abbas, O. M., & Mohammed, H. I. (2015). Prognosis and diagnosis of farm tractors reliability and availability for maintenance policies using Markov-chain model. *Universal Journal of Agricultural Research, 3*(4), 119–124. https://doi.org/10.13189/ujar.2015.030401

[20] Huang, L., Fu, Y., Mi, Y., Cao, J., & Wang, P. (2017). A Markov-chain-based availability model of offshore wind turbine considering accessibility problems. *IEEE Transactions on Sustainable Energy, 8*(4), 1592-1600. https://doi.org/10.1109/TSTE.2017.2695661

[21] Siddiqui, M. A., Butt, S. I., Gilani, O., Jamil, M., Maqsood, A., & Zhang, F. (2017). Optimizing availability of a framework in series configuration utilizing Markov model and Monte Carlo simulation techniques. *Symmetry, 9*(7), 96. https://doi.org/10.3390/sym9070096

[22] Mooren, M. (2018). *A Markov chain-based study on the availability of ship systems of a naval vessel: Insights for early-stage ship design requirements* [Master's thesis, Delft University of Technology]. TU Delft Repository. https://repository.tudelft.nl/record/uuid:05d4c1cd-b1be-4255-a5e3-c2e22dd7c09a

[23] Kumar, G., Jain, V., & Gandhi, O. P. (2018). Availability analysis of mechanical systems with condition-based maintenance using semi-Markov and evaluation of optimal condition monitoring interval. *Journal of Industrial Engineering International, 14*, 119-131. https://doi.org/10.1007/s40092-017-0212-z

[24] Tang, J., Ma, C., & Tian, P. (2020). Network availability evaluation based on Markov chain of QoS-aware. *Wireless Personal Communications, 113*, 1673-1689. https://doi.org/10.1007/s11277-020-07286-2

[25] Afsharnia, F., Marzban, A., Asoodar, M. A., & Abdeshahi, A. (2020). Availability modeling of sugarcane harvesting system by using Markov chain. *Journal of Biosystems Engineering, 45*, 145-154.

[26] Sun, A. K., Chang, H., Pullen, S., Kil, H., Seo, J., Morton, Y. J., & Lee, J. (2021). Markov chain-based stochastic modeling of deep signal fading: Availability assessment of dual-frequency GNSS-based aviation under ionospheric scintillation. *Space Weather, 19*, e2020SW002655. https://doi.org/10.1029/2020SW002655

[27] Sobhani, Z., & Shahrokhi, M. (2017). Optimization of availability of the systems with redundant components considering the half-state and deals. *Journal of Modeling in Engineering*, https://doi.org/10.22075/jme.2017.11373.1110.

[29] Sobhani, Z., & Shahrokhi, M. (2019). Availability optimization of a multi-state industrial system with the Markov chain approach. *Industrial Management Journal, 11*(3), 380-404. https://doi.org/10.22059/imj.2019.268528.1007516.

[30] Shahrokhi, M., & Sobhani, Z. (2018). Optimization of availability the systems with redundant considering the half-state, deals and repair rate's change. *Journal of Modeling in Engineering, 16*(54), 267-281. https://doi.org/10.22075/jme.2017.11373.1110.

[31] Shahrokhi, M., Sobhani, Z., & Bernard, A. (2021). Supplier selection by using the goal programming approach. In A. Dolgui, A. Bernard, D. Lemoine, G. von Cieminski, & D. Romero (Eds.), *Advances in production management systems. Artificial intelligence for sustainable and resilient production systems. APMS 2021* (IFIP Advances in Information and Communication Technology, Vol. 634, pp. 435-444). Springer. https://doi.org/10.1007/978-3-030-85914-5_47

[32] Gharegozlu, M., Sobhani, Z., & Farughi, H. (2018). Extended block diagram method for evaluating the reliability of multi-state system with dependent components. *Journal of Quality Engineering and Management, 8*(2), 75-85. https://dor.isc.ac/dor/20.1001.1.23221305.1397.8.2.1.5.

[33] Sobhani, Z. (2017). Redundancy optimization by considering inventory and lost production cost. *Journal of Quality Engineering and Management, 6*(4), 228-236.

[34] Sobhani, Z., Shahrokhi, M., & Bernard, A. (2023). An availability-based design optimization by using a fuzzy goal programming approach. *Journal of Industrial Engineering International, 19*(4), 18-32. https://www.proquest.com/openview/15c6d28cbba85574c461a0435b54164a.

[35] Sobhani, Z., Gharegozlu, M., Lin, J., & Farughi, H. (2025). Extended block diagram method for evaluating the reliability of multi-state system with dependent components. In *Proceedings of the IISE Annual Conference* (pp. 1–6). DOI:10.21872/2025IISE_6209

[36] Khojaste, A., Pearce, J., Pucci de Farias, D., Pritchard, G., & Zakeri, G. (2026). *Risk-averse Markov decision processes: Applications to electricity grid and reservoir management* (arXiv:2601.02207). arXiv. https://doi.org/10.48550/arXiv.2601.02207

[37] Khojaste, A., Pearce, J., Zakeri, G., & Sang, Y. (2025). *Electricity price-aware scheduling of data center cooling* (arXiv:2508.03160). arXiv. https://doi.org/10.48550/arXiv.2508.03160

[38] Basiri, M. A., Parvin, M., & Sobhani, Z. (2024). Implementing CRISP-DM and logistic regression for predictive analysis in financial transactions: A case study. *IRAIS 2024 Proceedings*, 6. https://aisel.aisnet.org/irais2024/6.

[39] Khojaste, A., Pritchard, G., & Zakeri, G. (2025). Quantile Fourier regressions for decision making under uncertainty. *Annals of Operations Research, 355*(3), 2843–2858. https://doi.org/10.1007/s10479-024-06306-9

[40] Kanagaraj, G., Ponnambalam, S. G., & Jawahar, N. (2016). Reliability-based total cost of ownership approach for supplier selection using cuckoo-inspired hybrid algorithm. *The International Journal of Advanced Manufacturing Technology, 84*(5–8), 801–816.

[41] Jiao, M., Tang, J., & Li, Y. (2008). Synthesis planning of parts importance in supplier involved product reliability design. In *2008 Chinese Control and Decision Conference* (pp. 2473–2476). https://doi.org/10.1109/CCDC.2008.4597769

[42] Kanagaraj, G., & Jawahar, N. (2009). A simulated annealing algorithm for optimal supplier selection using the reliability-based total cost of ownership model. *International Journal of Procurement Management, 2*(3), 244–266.

[43] Tomaru, T., Nakano, M. J., & Nishimura, H. (2011). Supplier quality assessment to identify depth technical knowledge of component reliability. *Production Planning & Control, 24*, 128–140.

[44] Hague, R. K., Barker, K., & Ramirez-Marquez, J. E. (2015). Interval-valued availability framework for supplier selection based on component importance. *International Journal of Production Research, 53*(20), 6083–6096. https://doi.org/10.1080/00207543.2015.1018454

[45] Yang, K. H. (2016). Supplier selection by considering cost and reliability. *International Journal of Economics and Management Engineering, 10*(7), 2292–2296.

[46] Rajabi Asadabadi, M. (2017). A customer based supplier selection process that combines quality function deployment, the analytic network process and a Markov chain. *European Journal of Operational Research, 263*(3), 1049–1062. https://doi.org/10.1016/j.ejor.2017.06.006

[47] Azarian, M. H., Das, D., & Pecht, M. (2020). Supplier selection. In *Center for Advanced Life Cycle Engineering (CALCE)*, University of Maryland, College Park, MD, USA (Chapter 11, pp. 197–205).

[48] Ho, W., Xu, X., & Dey, P. K. (2010). Multi-criteria decision making approaches for supplier evaluation and selection: A literature review. *European Journal of Operational Research, 202*(1), 16–24. https://doi.org/10.1016/j.ejor.2009.05.009

[49] Aissaoui, N., Haouari, M., & Hassini, E. (2007). Supplier selection and order lot sizing modeling: A review. *Computers & Operations Research, 34*(12), 3516–3540. https://doi.org/10.1016/j.cor.2006.01.016

[50] Qin, H., Luo, M., Gao, X., & Lim, A. (2012). The freight allocation problem with all-units quantity-based discount: A heuristic algorithm. *Omega, 40*(4), 415–423. https://doi.org/10.1016/j.omega.2011.05.005

[51] Hamdan, S., & Cheaitou, A. (2017). Dynamic green supplier selection and order allocation with quantity discounts and varying supplier availability. *Computers & Industrial Engineering, 110*, 573–589. https://doi.org/10.1016/j.cie.2017.03.028

[52] Pourmohammadreza, N., Jesri, Z. S. H., & Kamran, M. A. (2025). A review of discount policies in supplier selection: Trends, opportunities, and challenges. *IEEE Access, 13*, 43028–43055. https://doi.org/10.1109/ACCESS.2025.3547051

[53] Badi, I., Demir, G., Bouraima, M. B., & Maraka, A. K. (2025). Vendor managed inventory in practice: Efficient scheduling and delivery optimization. *Spectrum of Decision Making and Applications, 2*(1), 157–165.

[54] Gharibi, K., Hsieh, P.-H., Zhang, K., & Haapala, K. R. (2026). Toward sustainable industrial hemp value chains: Integrating real-world customer data and road networks within a mixed integer linear programming approach (Abstract ID 16576). IISE conference. https://doi.org/10.21872/2026IISE_16576

[55] Zimmermann, H.-J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems, 1*(1), 45–55. https://doi.org/10.1016/0165-0114(78)90031-3

[56] Kamran, M. A., Karimi, B., Bakhtiari, H., & Masoumzadeh, S. (2016). A resource allocation model in a healthcare emergency center using goal programming. *Journal of Engineering Research, 4*(4), 81–97. https://journals.ku.edu.kw/jer/index.php/JER/article/view/1172

[57] Masoumzadeh, S., Solimanpur, M., & Kamran, M. (2016). A multi-objective fuzzy goal programming P-hub location and protection model with back-up hubs considering hubs establishment fixed costs. *Scientia Iranica, 23*(4), 1941–1951. https://scientiairanica.sharif.edu/article_3939.html

[58] Çebi, F., & Otay, I. (2016). A two-stage fuzzy approach for supplier evaluation and order allocation problem with quantity discounts and lead time. *Information Sciences, 339*, 143–157. https://doi.org/10.1016/j.ins.2015.12.032

[59] Safaeian, M., Fathollahi-Fard, A. M., Tian, G., Li, Z., & Ke, H. (2019). A multi-objective supplier selection and order allocation through incremental discount in a fuzzy environment. *Journal of Intelligent & Fuzzy Systems, 37*(1), 1435–1455. https://doi.org/10.3233/JIFS-182843

[60] Gupta, S., Ali, I., & Ahmed, A. (2021). Multi-objective vendor selection problem of supply chain management under fuzzy environment. *Journal of the Operations Research Society of China, 9*(1), 33–62. https://doi.org/10.1007/s40305-018-0226-2

[61] Eslamipoor, R. (2022). A fuzzy multi-objective model for supplier selection to mitigate the impact of vehicle transportation gases and delivery time. *Journal of Data, Information and Management, 4*(3), 231–241. https://doi.org/10.1007/s42488-022-00076-4

[62] Keshavarz-Ghorabaee, M. (2023). Sustainable supplier selection and order allocation using an integrated ROG-based type-2 fuzzy decision-making approach. *Mathematics, 11*(9), 2014. https://doi.org/10.3390/math11092014

[63] Johnson, A. (2020). *Markov chain and its applications and introduction*. ISBN: 978-87-7022-096-5 (hardback), 978-87-7022-095-8 (e-book).
[64] Ross, S. M. (2019). *Introduction to probability models* (12th ed.). Academic Press.
[65] Mohammadi, H., Fazli, Z., Kaleh, H., Azimi, H. R., Moradi Hanifi, S., & Shafiee, N. (2021). Risk analysis and reliability assessment of overhead cranes using fault tree analysis integrated with Markov chain and fuzzy Bayesian networks. *Mathematical Problems in Engineering, 2021*, Article 6530541. https://doi.org/10.1155/2021/6530541